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Similarity of transport processes in disperse systems with suspended particles

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Abstract

On the basis of account for special features of hydrodynamics of disperse systems with suspended particles within the framework of the dimensional theory we obtained a system of dimensionless groups (35) which describes similarity of transport processes. System (35) involves new criteria $Fr_t^* = (u - u_t^*)/gH$, $\overline{J}_s^* = J_s/\rho_s(u - u_t^*)$ which involve an excess superficial gas velocity $u - u_t^*$ and are important generalized characteristics of convective motion of particles. Typical examples of use of system (35) for generalization of experimental data are presented. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

One of important stages of the design of large-scale industrial apparatuses with disperse media is study of transport processes on small-size laboratory setups at room temperatures and pressures. By virtue of this, the problem of scale transition which consists in the development of the rules of transfer (recalculation) of the results of laboratory experiments to a real apparatus is actual. Solution of this problem is based on analysis of similarity of transport processes in a disperse medium and construction of the system of a minimum number of dimensionless similarity criteria which are composed of independent characteristics of a specific system and make it possible to unambiguously determine the parameters of the laboratory setup simulating an industrial apparatus.

In the present paper, the problem mentioned is solved as applied to a wide class of disperse systems with suspended particles: fluidized bed, circulating fluidized bed, vertical pneumotransport. The possibility of combining them to one class is based on the existence of one important property. In all these systems, a particle weight is compensated by the force of gas friction and all excess power of the fan $\Delta p(u - u_t^*)$ is spent to accelerate suspended particles and create a complex pattern of convective (circulating) motion of them:

- one or several loops of internal circulation of particles (upward – in wakes of gas bubbles, downward – in the remaining emulsion phase) in a fluidized bed;
- one internal circulation loop (upward – in the bed core, downward – in the annular zone near the riser walls) in a circulating fluidized bed;
- upward convective motion of particles over the entire cross-section of the riser in vertical pneumotransport.

Intensity of large-scale motions of the solid phase is determined, probably, by an excess superficial gas velocity $u - u_t^*$. As will be shown below, this quantity plays an extremely important role in development of the rules of scale transition for disperse systems with suspended particles.

The problem of scaling in disperse systems have long attracted the attention of researchers and a number of attempts to solve it exists in literature. As a rule, the unknown system of dimensionless criteria was obtained as a result of one or another variant of nondimensionalization of the equations of balance of masses and impulses of phases.

Table 1 gives main systems of similarity criteria available in the literature [1–8]. An analysis of the results presented makes it possible to draw the following conclusions:

1. A number of dimensionless groups describing similarity of disperse systems amounts to 4–5. This gives a corresponding number of independent equations

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Nomenclature

$Ar = gd^3\rho_f(\rho_f - \rho_f)/\mu_f^2$	Archimedes number
c	specific heat capacity
d	particle diameter
D	bed (riser) diameter
D_f	coefficient of molecular diffusion of gas
D_t	disk diameter
$Fr = (u - u_{mf})^2/gH$, $Fr_D = (u - u_t)^2/gD$, $Fr_h = (u - u_{mf})^2/gh$, $Fr_t = (u - u_t)^2/gH$, $Fr_t^* = (u - u_t^*)^2/gH$	Froude numbers
g	gravity acceleration
h	height above the gas distributor
H	bed (riser) height
H_{mf}	bed height at $u = u_{mf}$
J_s	new specific solid mass flux
$\bar{J}_s = J_s/\rho_s(u - u_t)$, $\bar{J}_s^* = J_s/\rho_s(u - u_t^*)$	dimensionless solid mass fluxes
l	height of mass transfer probe
L	height or size of the bed
$Nu = \alpha d/\lambda_f$	Nusselt number
Δp	pressure drop
$Pr = c_f\mu_f/\lambda_f$	Prandtl number
r	radial coordinates
$Re = ud\rho_f/\mu_f$, $Re_t^* = u_t^* d\rho_f/\mu_f$	Reynolds numbers
S	cross-section of the riser
$Sc = \mu_f/\rho_f D_f$	Schmidt number
$Sh = \beta d/D_f$	Sherwood number
T	temperature
u	superficial gas velocity
u_b	bubble velocity
$v_b = u_b - (u - u_{mf})$	relative bubble velocity
u_t^*	velocity of particle floating under restricted conditions ($u_t^* \to u_{mf},\ \varepsilon \to \varepsilon_{mf}$, $u_t^* \to u_t,\ \varepsilon \to 1$).

Greek symbols

α	heat transfer coefficient
β	mass transfer coefficient
δ	width of a annular zone at riser walls
ε	porosity
θ	particle sphericity
λ	thermal conductivity
μ	dynamic viscosity
ρ	density
τ	shear stress on the riser wall
Φ	specific flow of particles
$\hat{\Phi}$	particle flow
σ	root-mean-square approximation error.

Subscripts

a	annular zone
c	core zone
cond	conductive
c–c	conduction–convection
f	gas
i	interphase
mf	minimum fluidization
s	particles
t	terminal conditions
w	at the riser wall

for determining parameters of a cold laboratory setup. It is obvious that this number of equations is insufficient for determination of seven unknown parameters:

J_s (or H_{mf}), u, d, D, H, ρ_s, c_s.

2. The systems of similarity criteria have narrow specialization, i.e., they are constructed as applied to a specific disperse system.
3. Some of the criteria are formed just formally without account for the specifics of disperse systems with suspended particles and do not have a clear physical meaning, which gives doubt about their correctness: u^2/gD, u^2/gd, u/u_{mf}, $J_s/\rho_s u$, d/D. The literature has no convincing proofs of the advantage of one or another system of similarity criteria.

The present paper is aimed at obtaining of a universal system of dimensionless similarity criteria which allow for the specifics of the considered disperse systems that make it possible to effectively generalize experimental data and unambiguously determine all parameters of the laboratory setup simulating a real apparatus.

2. Similarity of transport processes in disperse systems with suspended particles

Taking into account the above-mentioned specific features of the hydrodynamics of disperse systems with suspended particles, we can distinguish two main classes of transport processes which are characterized by different scales of velocity and length: (i) "macro" processes caused by circulating motion of particles with the characteristic scales $u - u_t^*$, D, and H; (ii) "micro" processes associated with interphase interaction "gas–particle" with the characteristic scales u_t^* and d.

We consider the problems of similarity of these processes in detail.

Table 1
Similarity criteria for disperse systems with suspended particles

Fluidized bed		Circulating fluidized bed		Vertical pneumotransport
$\frac{\rho_s-\rho_f}{\rho_f}, \frac{u_{mf}^2}{dg}, \frac{u_{mf} d\rho_f}{\mu_f}, \frac{H}{D}$	[1]	$\frac{u^2}{gD}, \frac{\rho_s}{\rho_f}, \frac{d}{D}, \frac{ud\rho_f}{\mu_f}, \frac{J_s}{\rho_s u}$	[4]	$\frac{J_s}{\rho_f u}, \frac{uD\rho_f}{\mu_f}, \frac{D}{d}, \frac{C_s}{C_f}$ [a]
$\frac{Lg}{u^2}, \frac{L}{d}, \frac{\rho_s}{\rho_f}, \frac{ud\rho_f}{\mu_f}$	[2]	$\frac{u^2}{gd\theta}, \frac{\rho_s}{\rho_f}, \frac{D}{d\theta}, Ar, \frac{J_s}{\rho_s u}$	[5]	
$\frac{D}{d}, \frac{L}{d}, \frac{\rho_f}{\rho_s}, Ar, \frac{u^2}{gd}$	[3]	$\frac{u^2}{gD}, \frac{\rho_s}{\rho_f}, \frac{u}{u_t}, \frac{J_s}{\rho_s u}$	[6]	
		$\frac{u^2}{gD}, \frac{\rho_s}{\rho_f}, \frac{u}{u_{mf}}, \frac{J_s}{\rho_s u}$	[7]	

[a] The system is composed of criteria used by different authors in generalization of experimental data [8].

2.1. Hydrodynamic processes caused by circulating motion of particles (macro processes)

The hydrodynamic characteristic of a disperse medium can be presented in the form of the function of the following parameters:

$$H = f\left(\left\{\begin{matrix} J_s \\ H_{mf} \end{matrix}\right\}, \rho_s, u - u_t^*, g, h, D, H\right). \quad (1)$$

The parameters J_s and H_{mf} which determine the mass of disperse material in the system are mutually eliminating: with account for H_{mf} (fluidized bed, circulating fluidized bed operating by the "furnace" scheme), when the pressure drop in the riser is specified $\Delta p = \rho_s(1-\varepsilon_{mf})gH_{mf}$, J_s is eliminated in (1). In contrast, when J_s is set (circulating fluidized bed operating by the "chemical reactor" scheme, vertical pneumotransport), H_{mf} is eliminated in (1).

Using the π-theorem of the dimensional theory [9], we write the dimensionless analog of (1)

$$H' = f\left(\left\{\begin{matrix} \overline{J}_s^* \\ H_{mf}/H \end{matrix}\right\}, Fr_t^*, \frac{h}{H}, \frac{H}{D}\right), \quad (2)$$

which, with account for the above, gives two equations:
1. fluidized bed and circulating fluidized bed operating by the furnace-type scheme:

$$H' = f_1\left(\frac{H_{mf}}{H}, Fr_t^*, \frac{h}{H}, \frac{H}{D}\right); \quad (3)$$

2. circulating fluidized bed operating by the chemical reactor-type scheme, vertical pneumotransport

$$H' = f_2\left(\overline{J}_s^*, Fr_t^*, \frac{h}{H}, \frac{H}{D}\right). \quad (4)$$

The obtained relations (3) and (4) allow one to establish the rates of scaling of hydrodynamic processes. These equations involve two generalized criteria of hydrodynamic similarity $Fr_t^* = (u-u_t^*)^2/gH$ and $\overline{J}_s^* = J_s/\rho_s(u-u_t^*)$. The Fr_t^* number characterizes the ratio of the kinetic energy of particles to their potential energy, the $\overline{J}_s^*$ number characterizes the concentration of particles in the system. We use specific examples and show that Eqs. (3) and (4) are in fact a general form of dimensionless relations generalizing experimental data on the hydrodynamics of disperse systems with suspended particles and establishing the scaling rules.

2.1.1. Fluidized bed

The equation for a diameter of gas bubbles

$$\frac{D_b}{h} = 1.3Fr^{1/3}\left(\frac{h}{H}\right)^{-1/3}, \quad (5)$$

which generalizes a great body of experimental data (more than 20 works), is obtained in [10]. To calculate a relative bubble velocity v_b, the authors of [11] found the formula

$$\frac{v_b}{u-u_{mf}} = 1.9Fr^{-1/3}\left(\frac{h}{H}\right)^{1/3}\left(\frac{H_{mf}}{D}\right)^{-1/2}. \quad (6)$$

In [12] it is suggested to calculate the bed expansion (concentration of particles) by the formula

$$\frac{\varepsilon-\varepsilon_{mf}}{1-\varepsilon} = 0.7Fr^{1/3}\left(\frac{H_{mf}}{H}\right)^{-1/3}\left(\frac{H_{mf}}{D}\right)^{1/2}. \quad (7)$$

2.1.2. Circulating fluidized bed

For a bed operating by the chemical reactor scheme the authors of [13] derived an extremely simple equation for calculating the concentration of particles over the height of the transport zone:

$$1-\varepsilon = \frac{\rho}{\rho_s} = \overline{J}_s\left(\frac{h}{H}\right)^{-0.82}, \quad (8)$$

which generalizes a great body of experimental data (10 works).

According to general dependence (4), we generalized the data on measurement of downward particle mass fluxes at the riser wall [14–20]

$$\frac{\Phi_w}{\rho_s(u-u_t)} = 0.8\,\bar{J}_s Fr_t^{-1.2}\left(\frac{h}{H}\right)^{-0.82}\left(\frac{D}{H}\right)^{0.67}. \tag{9}$$

The experimental points and correlation (9) are shown in Fig. 1. A root-mean-square error of approximation is 27%. The ranges of change of the bed characteristics are: $7 \leqslant H \leqslant 33.5$ m, $0.144 \leqslant D \leqslant 4.7$ m. $0.22 \leqslant h/H \leqslant 0.92$, $4.2 \leqslant J_s \leqslant 42.4$ kg/m^2 s, $0.003 \leqslant Fr_t \leqslant 0.25$. For rectangular risers $D = \sqrt{4S/\pi}$, for $S = 12 \times 4.7$ m D was taken equal to 4.7 m. We note that an analysis of experimental data in beds operating by the furnace scheme necessitates preliminary establishment of the dependence of the external mass flow J_s on the determining factors

$$\bar{J}_s = \varphi\left(Fr_t, \frac{H_{mf}}{H}, \frac{H}{D}\right), \tag{10}$$

which makes it possible to pass from Eqs. (3) and (4). Generalization of experimental data [21,22] by (10) gave the equation

$$\bar{J}_s = 0.54 Fr_t^{0.8}\left(\frac{H_{mf}}{H}\right)^{0.8}, \quad \sigma = 12\%$$
$$(0.1 \leqslant H_{mf} \leqslant 0.5 \text{ m},\ 0.2 \leqslant d \leqslant 0.32 \text{ mm},$$
$$2.0 \leqslant u \leqslant 6.5 \text{ m/s},\ 6.6 \leqslant H \leqslant 13.5 \text{ m}). \tag{11}$$

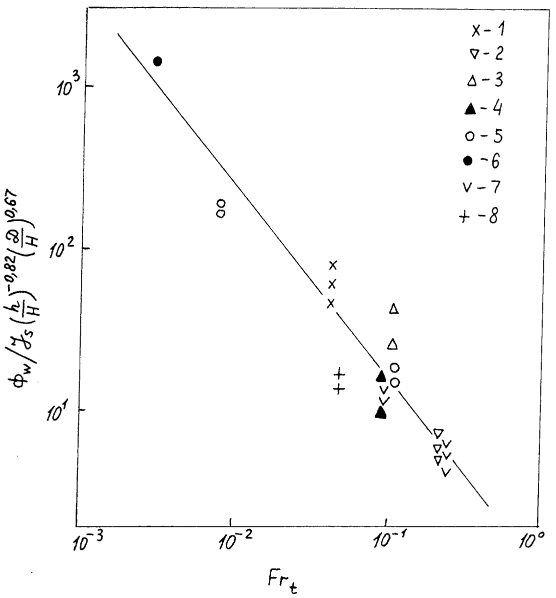

Fig. 1. Descending mass fluxes of particles at the riser wall: (1) [14], $S = 0.7 \times 0.118$ m; (2) [15], $D = 0.144$ m; (3) [16], $S = 0.286 \times 0.176$ m; (4) [17], $S = 0.8 \times 1.2$ m; (5) [18], $S = 1.72 \times 1.44$; (6) [18], $S = 12 \times 4.7$ m; (7) [19], $D = 0.15$ m; (8) [20], $D = 0.161$ m.

Eq. (9) can be presented in a more compact form, if dependence (8) is taken into account

$$\frac{\Phi_w}{\rho_s(u-u_t)} = 0.8\frac{\rho}{\rho_s} Fr_t^{-1.2}\left(\frac{D}{H}\right)^{0.67}. \tag{12}$$

Eq. (12) indicates direct dependence of the dimensionless particle mass flux at the riser wall on a mean concentration of them in the horizontal cross-section.

The authors of [18] obtained the empirical dependence of the width of the annular zone at the riser wall on its diameter

$$\delta = 0.05 D^{0.74}, \quad 0.07 \leqslant D \leqslant 8 \text{ m}. \tag{13}$$

Assuming independence of δ of the height above the gas distributor and of the superficial gas velocity, we can write (13) in a dimensionless form

$$\frac{\delta}{D} = 0.025\left(\frac{H}{D}\right)^{0.26}, \quad H \leqslant 33.5 \text{ m}. \tag{14}$$

Allowing for relative smallness of δ, we can assume that a local downward particle flux linearly depends on the radial coordinate

$$\Phi_a(r) = \Phi_w\left(1 - \frac{D}{2\delta} + \frac{r}{\delta}\right). \tag{15}$$

Function (15) satisfies the conditions

$$\Phi_a\left(\frac{D}{2}\right) = \Phi_w, \quad \Phi_a\left(\frac{D}{2} - \delta\right) = 0.$$

For an integral downward flow at the wall

$$\hat{\Phi}_a = 2\pi \int_{D/2-\delta}^{D/2} \Phi(r) r\,dr = \frac{\pi D \delta}{2}\Phi_w. \tag{16}$$

With account for (9) and (14) we obtain the formula for calculating $\hat{\Phi}_a$:

$$\hat{\Phi}_a \Big/ \frac{\pi D^2}{4} J_s = 0.04 Fr_t^{-1.2}\left(\frac{h}{H}\right)^{-0.82}\left(\frac{D}{H}\right)^{0.41}. \tag{17}$$

On the basis of (17) we can easily obtain the calculation relation for the upward particle flux in the core zone. We use the balance equation which determine $\hat{\Phi}_c$:

$$\hat{\Phi}_c = \frac{\pi D^2}{4} J_s + \hat{\Phi}_a. \tag{18}$$

Substitution of $\hat{\Phi}_a$ from (17) into (18) yields the unknown equation

$$\hat{\Phi}_c \Big/ \frac{\pi D^2}{4} J_s = 1 + 0.04 Fr_t^{-1.2}\left(\frac{h}{H}\right)^{-0.82}\left(\frac{D}{H}\right)^{0.41}. \tag{19}$$

2.1.3. Vertical pneumotransport

In this case, Eq. (4) holds. It is simplified due to the absence of internal circulation of particles in the system (the parameters h and H drop out of the determining ones):

$$H' = f_2(\bar{J}_s^*, Fr_D). \tag{20}$$

Generalization of experimental data [23,24] on measurement of shear stress on the riser walls led to a simple equation:

$$\frac{\tau}{\rho_f u^2} = 0.17\sqrt{\bar{J}_s}, \tag{21}$$

which is shown in Fig. 2. The region of verification of (21) is: $1-\varepsilon \leqslant 2\times 10^{-2}$, $0.05 \leqslant D \geqslant 0.18$ m, $4.3 \leqslant u \leqslant 20$ m/s, $12 \leqslant J_s \leqslant 434$ kg/m^2 s, $0.06 \leqslant d \leqslant 1.18$ mm.

It can easily be shown that the concentration of particles is calculated by the formula (see Fig. 3)

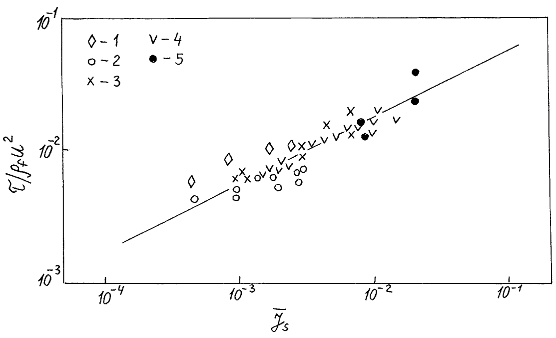

Fig. 2. Tangential stress in vertical two-phase flows: (1) $d = 0.113$ mm; (2) 0.1; (3) 0.2; (4) 1.18; (5) 0.06; (1)–(4) [24]; (5) [23].

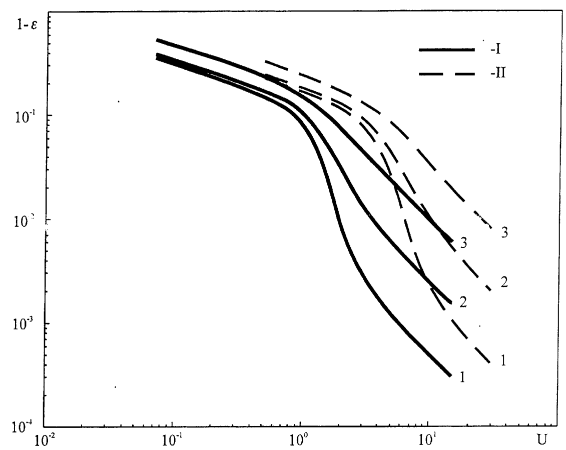

Fig. 3. Dependence of volumetric particle concentration on the superficial gas velocity: (I) $\rho_s = 2600$ kg/m^3, $d = 0.2\times 10^{-3}$ m; $Ar = 770$, $u_t = 1.66$ m/s; $u_{mf} = 0.05$ m/s; (II) $\rho_s = 1089$ kg/m^3, $d = 1.67\times 10^{-3}$ m, $Ar = 184{,}094$, $u_t = 5.9$ m/s, $u_{mf} = 0.45$ m/s, (1) $J_s = 10$ kg/m^2 s; (2) 50; (3) 200.

$$\frac{\rho}{\rho_s} = 1-\varepsilon = \bar{J}_s^*/(1+\bar{J}_s^*). \tag{22}$$

In fact, the quantity J_s is determined by the relation

$$J_s = \rho_s(1-\varepsilon)v = \rho v. \tag{23}$$

Substitution of the expression for the particle velocity $v = (u-u_t^*)/\varepsilon$ into (23) gives the formula:

$$\frac{\rho}{\rho_s} = \frac{J_s}{\rho_s(u-u_t^*)+J_s}, \tag{24}$$

which is likely the dimensional form of (22). Since the number $\bar{J}_s^*$ is the function of ε (the velocity u_t^* depends on ε), formula (22) is the transcendental equation with respect to ε. Fig. 4 shows the solutions of (22) for different values of Ar, u, and J_s. In the case of a rarefied system ($1-\varepsilon \leqslant 0.01$) $\bar{J}_s^* \approx \bar{J}_s \ll 1$ and Eq. (22) is simplified

$$\frac{\rho}{\rho_s} = 1-\varepsilon = \bar{J}_s. \tag{25}$$

Experimental data on values of the near-wall mass flows of particles in the riser with a diameter of 0.18 m are given in [23]. Processing of these data by (20) led to simple equations:

1. the absence of particle heap at the riser wall:

$$\frac{\Phi_w}{\rho_s(u-u_t^*)} = 0.03\bar{J}_s^{0.35}, \quad \sigma = 6\%$$
$$(10 \leqslant u \leqslant 13.6 \text{ m/s},\ 152 \leqslant J_s \leqslant 434 \text{ kg/m}^2 \text{ s},\ d = 0.06 \text{ mm}); \tag{26a}$$

2. the presence of particle heap (downward particle flow) at the riser wall:

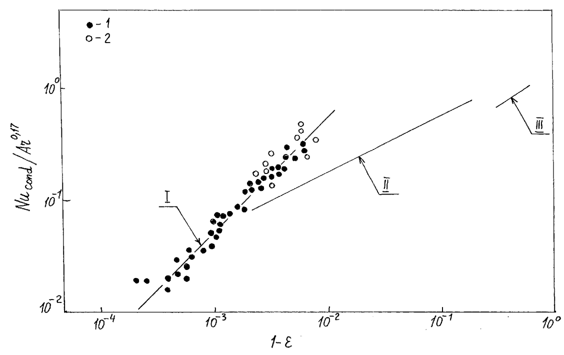

Fig. 4. Conductive–convective heat transfer in disperse systems with suspended particles: (1) [31], $d = 0.076$ mm; (2) [8], $d = 0.14$–0.165 mm. (I) $Nu_{cond} = 54.0Ar^{0.17}(1-\varepsilon)$ from (40); (II) $Nu_{cond} = 1.83Ar^{0.17}(1-\varepsilon)^{0.50}$ from (36b); $Ar = 200$; (III) $Nu_{cond} = 1.40Ar^{0.17}(1-\varepsilon)^{0.66}$ from (36). $Ar = 200$; $c_s = 800$ J/kg K; $\rho_s = 2500$ kg/m^3.

$$\frac{\Phi_{\rm w}}{\rho_{\rm s}(u-u_{\rm t}^*)} = 184\bar{J}_{\rm s}Fr_{\rm D}^{-1.7}, \quad \sigma = 23\%$$
$$(4.3 \leqslant u \leqslant 11.0\ {\rm m/s},\ 133 \leqslant J_{\rm s} \leqslant 514\ {\rm kg/m^2\ s}). \tag{26b}$$

2.2. Process of interphase heat and mass transfer (micro processes)

In this case, the system of dimensional determining parameters has the form:

$$u_{\rm t}^*, d, \rho_{\rm f}, \rho_{\rm s}, g, \mu_{\rm f}, D_{\rm f}, \lambda_{\rm f}, c_{\rm f}, c_{\rm s}. \tag{27}$$

The dimensionless characteristic of the process is:
1. heat transfer

$$Nu_{\rm i} = \varphi\left(Re_{\rm t}^*, Ar, Pr, \frac{c_{\rm s}}{c_{\rm f}}, \frac{\rho_{\rm s}}{\rho_{\rm f}}\right); \tag{28}$$

2. mass transfer

$$Sh_{\rm i} = \varphi\left(Re_{\rm t}^*, Ar, Sc, \frac{\rho_{\rm s}}{\rho_{\rm f}}\right). \tag{29}$$

We note that in that cases $u_{\rm t}^* \to u_{\rm mf}$ and $u_{\rm t}^* \to u_t$, unambiguous connection exists between the numbers Ar and $Re_{\rm t}^*$ and in (28) and (29) we can use one of the two numbers ($Re_{\rm t}^*$ and Ar) instead of both.

2.2.1. Fluidized bed

To calculate interphase heat transfer in the region of "true" coefficients of heat transfer ($Re/\varepsilon > 200$), using the relation $Re_{\rm mf} = 0.25Ar^{0.5}$ which holds in the region of high Ar, the authors of [25] obtained

$$Nu_{\rm i} = 0.26(ArPr)^{1/3}, \tag{30}$$

In [26], for the coefficient of heat transfer of a moving particle of diameter $D_{\rm s} > d$, the equation

$$Nu_{\rm i} = \frac{1}{K_0}\left(\frac{D_{\rm s}}{d}\right)^{-0.2}\left(0.53Ar^{0.1}\left(\frac{\rho_{\rm s}}{\rho_{\rm f}}\right)^{0.14}\left(\frac{c_{\rm s}}{c_{\rm f}}\right)^{0.24} + 0.058Ar/(1400 + 5.22\sqrt{Ar})\right), \tag{31}$$

which holds in beds under pressure ($0.1 \leqslant p \leqslant 10.0$ MPa), $K_0 = -0.4$–0.6 (depending on the Ar), was obtained.

2.2.2. Vertical pneumotransport

For the case of small concentrations of particles ($1-\varepsilon < 0.35 \times 10^{-3}$) it was obtained [8]:

$$Nu_{\rm i} = 0.186Re_{\rm t}^{0.8} \quad (30 \leqslant Re_{\rm t} < 480); \tag{32a}$$
$$Nu_{\rm i} = 1.14Re_{\rm t}^{0.5} \quad (480 < Re_{\rm t} \leqslant 2000). \tag{32b}$$

2.3. Processes of exchange between a disperse bed and motionless surfaces (general case)

Intensity of the exchange processes in this case is determined by the effect of both macro- and micro-processes analyzed above. Therefore, the following system:

$$\left\{\begin{matrix} J_{\rm s} \\ H_{\rm mf} \end{matrix}\right\}, u-u_{\rm t}^*, D, H, h, g, \rho_{\rm f}, \rho_{\rm s}, c_{\rm f}, c_{\rm s}, u_{\rm t}^*, d, \mu_{\rm f}, D_{\rm f}, \lambda_{\rm f}, u, \tag{33}$$

which amounts to the combination of parameters (1) and (27), must be taken as determining parameters. The velocity $u_{\rm t}^*$ is unambiguously determined quantities $\rho_{\rm f}$, $\rho_{\rm s}$, ε, d, $\mu_{\rm f}$, and g and it can be eliminated from (33). Using the π-theorem of the dimensional theory, we obtain the system of dimensionless groups

$$\left\{\begin{matrix} \bar{J}_{\rm s}^* \\ H_{\rm mf}/H \end{matrix}\right\}, Fr_{\rm t}^*, Ar, Re, Pr, Sc, \frac{h}{H}, \frac{H}{D}, \frac{c_{\rm s}}{c_{\rm f}}, \frac{\rho_{\rm s}}{\rho_{\rm f}}, \frac{\rm d}{D}. \tag{34}$$

The simplex d/D, which is the ratio of micro- and macro-scales of length, can be eliminated from system (34) as formal and is physically senseless. Then, system (34) is

$$\left\{\begin{matrix} \bar{J}_{\rm s}^* \\ H_{\rm mf}/H \end{matrix}\right\}, Fr_{\rm t}^*, Ar, Re, Pr, Sc, \frac{h}{H}, \frac{H}{D}, \frac{c_{\rm s}}{c_{\rm f}}, \frac{\rho_{\rm s}}{\rho_{\rm f}}. \tag{35}$$

2.3.1. Fluidized bed

In [27], on the basis of numerous experimental data on conduction–convection heat exchange between a bed and immersed surfaces, a semi-empirical formula

$$Nu_{\rm c\text{-}c} = 0.74Ar^{0.1}(1-\varepsilon)^{2/3}\left(\frac{c_{\rm s}}{c_{\rm f}}\right)^{0.24}\left(\frac{\rho_{\rm s}}{\rho_{\rm f}}\right)^{0.14} + 0.046\frac{Re}{\varepsilon}Pr(1-\varepsilon)^{2/3} \tag{36}$$

is obtained, where ε is given by (7). Relation (36) is checked within a very wide range of variation of experimental conditions: $1.4 \times 10^2 \leqslant Ar \leqslant 1.1 \times 10^7$, $0.1 \leqslant p \leqslant 10.0$ MPa. This makes it a most universal formula known in the literature.

In generalization of experimental data on mass transfer of a vertical naphthalene plate of thickness 10 mm given by Suprun [28], we found the relation

$$Sh = 0.76Ar^{0.20}Re^{0.30}\left(\frac{l}{d}\right)^{-0.5}Sc^{0.33},$$
$$\sigma = 13\%\ (0.32 \leqslant d \leqslant 2.5\ {\rm mm},\ 0.04 \leqslant l \leqslant 0.2\ {\rm m}). \tag{37}$$

Using the local analog of the Fr number, we obtained the formula which describes the systematic data of Michkovskii [29] on the value of force affecting a horizontal disc immersed in the fluidized bed:

$$\frac{F}{\rho_s g D_t^3} = 0.3Ar^{-0.17}Fr_h^{0.5}\left(\frac{h}{D_t}\right)^{1.5}, \quad \sigma = 21\%$$
$$(0.02 \leqslant D_t \leqslant 0.08 \text{ m}, \ H_{mf} \geqslant 0.5 \text{ m}, \ 0.074 \leqslant d \leqslant 0.695 \text{ mm}). \tag{38}$$

We note that to generalize these data the authors of [29] used the number $Fr_d = (u - u_{mf})^2/gd$ composed of the characteristics of large-$(u - u_{mf})$ and small-scale (d) transport processes. It is obvious that this number is physically senseless.

2.3.2. Circulating fluidized bed

To calculate a conduction–convection component of the heat transfer coefficient, in [30] we suggested the equation

$$Nu_{c\text{–}c} = 1.65Ar^{0.12}\bar{J}_s^{0.5}\left(\frac{h}{H}\right)^{-0.41} + 0.00049Ar^{0.69}Pr, \tag{39a}$$

Taking into account formula (8), we can present (39a) in the form

$$Nu_{c\text{–}c} = 1.65Ar^{0.19}(1-\varepsilon)^{0.5} + 0.00049Ar^{0.69}Pr, \tag{39b}$$

The formula was verified under the conditions: $0.1 \leqslant p \leqslant 5$ MPa, $0.058 \leqslant d \leqslant 0.827$ mm, $1 - \varepsilon \leqslant 0.2$. Due to specific features of the hydrodynamics of the bed at the riser wall (descending flow of particles and gas filtration at a velocity much smaller than a superficial velocity), the number Ar substituted the number Re in the convective component $Nu_{c\text{–}c}$.

2.3.3. Vertical pneumotransport

As a result of generalization of experimental data of [8,31], we obtained the formula for calculating $Nu_{c\text{–}c}$, which is similar to (36) and (39a), (39b):

$$Nu_{c\text{–}c} = 54.0Ar^{0.17}(1-\varepsilon) + 0.021Re^{0.8}\left(\frac{d}{D}\right)^{0.2}Pr^{0.43}, \quad \sigma = 8\%, \tag{40}$$

where $1 - \varepsilon = \bar{J}_s$. The region of verification of (40) is: $\bar{J}_s \leqslant 10^{-2}$, $0.076 \leqslant d \leqslant 0.15$ mm, $12 \leqslant u \leqslant 42$ m/s.

Fig. 4 presents experimental data [8,31] and function (40). The same figure shows functions (36) and (39a), (39b). To be correct, we compare the conductive components of $\alpha_{c\text{–}c}$ for the case $Ar = 200$. An analysis of Fig. 4 makes it possible to draw the following conclusions:

1. as $1 - \varepsilon$ increases the values of α_{cond} for a circulating fluidized bed asymptotically approach the values of α_{cond} for a fluidized bed;
2. at the same values of $1 - \varepsilon$, α_{cond} for a circulating fluidized bed is about 1.5–2.5 times lower than for pneumotransport.

3. Scale transition in disperse systems with suspended particles (similarity of disperse beds)

The obtained system (35) characterizes similarity of local transport processes. Similarity of integral process is achieved at the corresponding equality of dimensionless parameters:

$$\left\{\begin{matrix} J_s^* \\ H_{mf}/H \end{matrix}\right\}, Fr_t^*, Ar, Re, Pr, Sc, \frac{D}{H}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}. \tag{41}$$

Seven complexes of system (41)

$$\left\{\begin{matrix} J_s^* \\ H_{mf}/H \end{matrix}\right\}, Fr_t^*, Ar, Re, \frac{D}{H}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}. \tag{42}$$

Give seven independent equations for determining seven parameters of the cold laboratory setup simulating an industrial apparatus

Table 2
Simulation of a furnace and a gas generator with a circulating fluidized bed operating by the chemical reactor-type scheme

Quantity	Similarity			
	Furnace $T = 800$°C, $p = 0.1$ MPa	Laboratory setup	Gas generator $T = 1000$°C, $p = 2$ MPa	Laboratory setup
J_s, kg/m^2 s	50.0	83.2	50.0	15.8
H, m	12.0	2.2	12.0	19.1
D, m	1.0	0.18	1.0	1.59
u, m/s	6.0	2.57	6.0	7.6
d, m	0.2×10^{-3}	0.048×10^{-3}	0.2×10^{-3}	0.31×10^{-3}
ρ_s, kg/m^3	2600	10064	2600	650
c_s, J/kg °C	730	660	730	610
ρ_f, kg/m^3	0.31	1.2	4.81	1.2
μ_f, kg/m s	449×10^{-7}	179×10^{-7}	365×10^{-7}	179×10^{-7}
u_t, m/s	1.06	0.45	0.81	1.03

Table 3
Simulation of a furnace and a gas generator with a circulating fluidized bed operating by the furnace-type scheme

Quantity	Similarity			
	Furnace $T = 800°C$, $p = 0.1$ MPa	Laboratory setup	Gas generator $T = 1000°C$, $p = 2$ MPa	Laboratory setup
H_{mf}, m	0.5	0.092	0.5	0.8
H, m	12.0	2.2	12.0	19.2
D, m	1.0	0.18	1.0	1.59
u, m/s	6.0	2.57	6.0	7.6
d, m	0.2×10^{-3}	0.048×10^{-3}	0.2×10^{-3}	0.31×10^{-3}
ρ_s, kg/m^3	2600	10064	2600	650
c_s, J/kg °C	730	660	730	610

$$\left\{ \begin{array}{c} J_s \\ H_{mf} \end{array} \right\}, d, D, H, u, \rho_s, c_s. \quad (43)$$

Tables 2 and 3 give the results of calculation of parameters of laboratory circulating fluidized beds ($T = 20°C$, $p = 0.1$ MPa, a fluidizing medium is air) on the basis of (42) which simulate conduction–convection transport process in high-temperature installations operating following different schemes of control over the amount of disperse material in the riser.

4. Conclusions

On the basis of the analysis of specific features of the hydrodynamics of disperse systems with suspended particles within the framework of the dimensional theory we obtained the complete system of dimensionless groups (35) which describes similarity of transport processes in the media mentioned. Seven complexes of system (35)

$$\left\{ \begin{array}{c} \bar{J}_s^* \\ H_{mf}/H \end{array} \right\}, Fr_t^*, Ar, Re, \frac{D}{H}, \frac{c_s}{c_f}, \frac{\rho_s}{\rho_f}$$

allow one to find seven parameters of the cold laboratory setup simulating the industrial apparatus and thus to determine the sought scaling laws. The complexes Fr_t^* and $\bar{J}_s^*$ introduced on the basis of an excess superficial gas velocity $u - u_t^*$ are important generalized characteristics of a convective (circulating) flow of particles in these disperse media. Computation relations obtained by these numbers are rather simple and possess, as a rule, a high degree of universality. This makes them very convenient for use in engineering practice.

The accumulated experience of use of the numbers Fr_t^* and $\bar{J}_s^*$ in combination with other criteria of system (35) allows one to speak of the development of a new effective technique of generalization of experimental data in disperse systems with suspended particles. Use of this technique makes it possible, in many cases, to turn the process of generalization of experimental data to a simple and routine work.

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